

1. NO CALCULATORS OR NOTES ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

A certain economist believes that the rate at which a person's wealth changes is proportional to the difference between their country's median wealth and their own wealth. Assuming that median wealth is a constant (W_{MEDIAN}), and that wealthy people (people with a lot of wealth) tend to get wealthier, write a differential equation for the wealth $W(t)$ of a wealthy person at time t .

SCORE: 3 / 3 PTS

NOTE: The sign of all constants should be stated clearly.

$$\frac{dW}{dt} = k(W(t) - W_{\text{median}}) \quad (2)$$

constant of proportionality

k is positive (1)

What does the Existence and Uniqueness Theorem tell you about possible solutions to the initial value problem $(\frac{dy}{dx})^3 + 1 = y$, $y(3) = 1$? **Justify your answer properly, but briefly.**

SCORE: / 4 PTS

$$\frac{dy}{dx} = f(y) = \sqrt[3]{y-1} \quad (1)$$

exists \leftarrow

$f(y)$ is continuous at $(3, 1)$

not unique \leftarrow

$f'(y) = \frac{1}{3}(y-1)^{-2/3}$ is not continuous around $(3, 1)$

\therefore we... have learned little

There exists a soln. @ $y(3) = 1$,
but it may not be unique
(possibly other solns)

Consider the IVP $y' = x(y-1)$, $y(2) = 6$. Use Euler's method with $h = 0.1$ to estimate $y(2.2)$.

SCORE: / 4 PTS

x	y	y'
2	6	10
2.1	3	4.2
2.2	3.42	

$$y'(2) = 2(6-1) = 10$$

$$y(2.1) = 2 + 0.1(10) = 2 + 1$$

$$y'(3) = 2.1(2) = 4.2$$

$$y(2.2) = 3 + 0.1(4.2) = 3 + 0.42$$

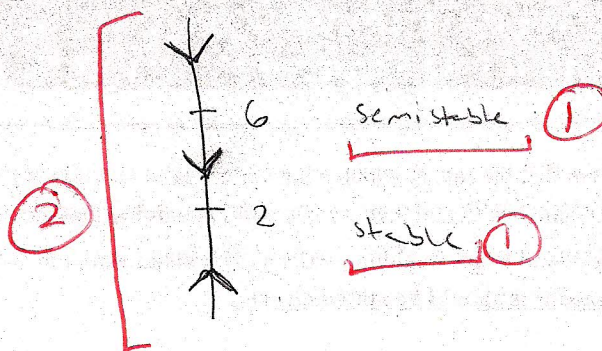
$$y(2.2) \approx 3.42$$

Consider the autonomous DE $y' = (2 - y)^3(6 - y)^2$.

SCORE: 6 / 6 PTS

- [a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.
You must draw a phase portrait to get full credit.

$$y' = 0 \Leftrightarrow y = 2 \quad y = 6$$



- [b] If $y = f(x)$ is a solution of the DE such that $f(7) = 1$, what is $\lim_{x \rightarrow \infty} f(x)$?



2 (1)

- [c] If $y = g(x)$ is a solution of the DE such that $g(8) = 5$, what is $\lim_{x \rightarrow \infty} g(x)$?



2 (1)

Consider the DE $x^2 y'' - xy' + y = \sqrt{x}$.

SCORE: ____ / 7 PTS

- [a] Is $y = 4\sqrt{x} + Ax + Bx \ln x$ a family of solutions of the DE?

$$y' = 2x^{-1/2} + A + B(1 + \ln x + \frac{x}{x})$$

$$= 2x^{-1/2} + A + B \ln x + B$$

$$y'' = -x^{-3/2} + \frac{B}{x}$$

~~*~~

$$= x^2 (-x^{-3/2} + Bx^{-1}) - x (2x^{-1/2} + B \ln x + A + B) + (4x^{1/2} + Bx \ln x + Ax + Bx)$$

$$= x^{1/2} + Bx^{-1} - Bx$$

- [b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = 6$, $y'(1) = 2$.
 If the answer to [a] is "NO", write "SKIP" and skip this part.

skip? Δ

[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS:

[1]

d ✓

[2]

e ✓

[3]

d ✓

[4]

f ✓

[5]

c ✓

[6]

a ✓

[1]

How much of your learning does the instructor believe comes from your daily reading and homework combined ?

[a]

40%

[b]

50%

[c]

60%

[d]

70%

[e]

80%

[2]

Which statement below regarding tests (quizzes, midterms, final exam) is false ?

[a]

If you continue writing on your test after the stated ending time, you will receive a 0 for that test.

[b]

There are no make-ups for missed quizzes.

[c]

The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.

[d]

If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.

[e]

If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

[3]

Proper use of the textbook for this class includes

[a]

understanding all the terminology used in the book

[b]

working out the given examples yourself and checking that you are able to get the same results as the book

[c]

reading the sections of the textbook before the corresponding lecture

[d]

all of the previous answers [a], [b] and [c]

[e]

some, but not all, of the previous answers [a], [b] and [c]

[4]

If you score 120 points on Midterm 1, 140 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

(HINT: You are encouraged to start studying regularly early in the quarter.)

[a]

Midterm 1's score will be changed to 145 (the highest midterm score)

[b]

Midterm 1's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)

[c]

Midterm 1's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 1's and Midterm 2's scores)

[d]

Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 1's and the highest midterm's scores)

[e]

Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ and Midterm 2's score will be changed to $(140 + 145) \div 2 = 142.5$ (the average of each midterm's and the highest midterm's score)

[f]

no midterm scores will be changed

[5]

If you score 140 points on Midterm 1, 120 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

[a]

Midterm 2's score will be changed to 145 (the highest midterm score)

[b]

Midterm 2's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)

[c]

Midterm 2's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 2's and Midterm 1's scores)

[d]

Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 2's and the highest midterm's scores)

[e]

Midterm 1's score will be changed to $(140 + 145) \div 2 = 142.5$ and Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of each midterm's and the highest midterm's score)

[f]

no midterm scores will be changed

[6]

Which statement below regarding attendance is false ?

[a]

Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.

[b]

Arriving late on a quiz or midterm day will not be counted as late.

[c]

Unexcused early departures are considered absences.

[d]

If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.

[e]

Attendance policies will not apply to you if you score more than 80% on every midterm.